RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SIXTH SEMESTER EXAMINATION, MAY 2018
THIRD YEAR [BATCH 2015-18]
MATHEMATICS (Honours)
Paper : VIII

Date : 07/05/2018
Time : 11 am – 2 pm
Full Marks : 70

[Use a separate Answer Book for each Group]

Group - A

(*Answer* ***any five*** *questions*) [5×6]

1. The square frame ABCD of four equal jointed rods hangs from A, the shape being maintained by a string joining midpoints of AB & BC. Prove that the ratio of the tension of the string to the reaction at C is $\frac{8}{\sqrt{5}}$.

2. A ladder whose C.G. divides it into two portions of lengths a & b, rests with one end on a rough horizontal floor & the other end against a rough vertical wall. If the co-efficient of friction at the floor & the wall be μ & μ' respectively, show that the inclination θ of the ladder to the floor, when the equilibrium is limiting, is given by $\tan\theta = \frac{a - b\mu\mu'}{(a+b)\mu}$.

3. A given length 2s of a uniform chain has to be hang between two points at the same level and the tension has not to exceed the weight of length b of the chain. Show that the greatest span of the chain is $\sqrt{b^2 - s^2}\,\log_e\left[\frac{b+s}{b-s}\right]$.

4. A solid hemisphere is supported by a string fixed to point on its rim & to a point on a smooth vertical wall with which the curved surface of the sphere is in contact. If θ & φ are the inclinations of the string & the plane base of the hemisphere to the vertical then using principle of virtual work prove that $\tan\varphi = \frac{3}{8} + \tan\theta$.

5. A smooth paraboloid of revolution is fixed with its axis vertical and vertex upwards; on it is placed a heavy elastic string of unstretched length $2\pi c$; when the string is in equilibrium show that it rests in the form of a circle of radius $\frac{4\pi a c\lambda}{4\pi a\lambda - cW}$, where W is the weight of the string, λ, its modulus of elasticity and 4a, the latus rectum of the generating parabola.

6. A body consisting of a cone and a hemisphere on the same base, rests on a rough horizontal table, the hemisphere being in contact with the table; show that the greatest height of the cone, so that the equilibrium may be stable, is $\sqrt{3}$ times the radius of the hemisphere.

7. A lamina in the form of an isosceles triangle, whose vertical angle α is placed on a sphere of radius r so that its plane is vertical & one of its equal sides is in contact with sphere. Show that if the triangle be slightly displaced in its own plane the equilibrium will be stable if $\sin\alpha < \frac{3r}{a}$ where a is the length of the equal sides of the triangle.

8. Forces X, Y, Z act along the three straight lines $y = b$, $z = -c$; $z = c$, $x = -a$; $x = a$, $y = -b$ respectively; show that they will have a single resultant if $\frac{a}{X} + \frac{b}{Y} + \frac{c}{Z} = 0$ and the equations of its line of action are any two of the three $\frac{y}{Y} - \frac{z}{Z} - \frac{a}{X} = 0$, $\frac{z}{Z} - \frac{x}{X} - \frac{b}{Y} = 0$, $\frac{x}{X} - \frac{y}{Y} - \frac{c}{Z} = 0$.

Group - B

(*Answer **any two** questions*) [2×10]

9. a) Write a suitable program in C to find the sum of the infinite series $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\ldots$ correct to six decimal places for $x=0{\cdot}25$. [3]

b) In a Boolean Algebra $(B,+,\bullet,')$ prove that—

i) $(a+b)'=a'\cdot b'$ [2]

ii) $a+b'=1$ iff $a+b=a$. [2]

c) Describe the output generated by the following program. [3]

```
#  include <stdio.h>
   int a = 100, b = 200;
   int funct1 (int count);
   int funct2 (int i);
main ( )
   {
      int count;
      for (count = 1; count < = 5; ++ count)
            printf ("% d  ", funct1 (count));
      printf ("\n");
   }
int funct1 (int x)
   {
      int c, d;
      c = funct2(x);
      d = (c < 100)? (a+c) : b;
      return (d);
   }
int funct 2 (int x)
   {
      static int prod = 1;
      prod * = x;
      return prod;
   }
```

10. a) A C program contains the following declaration : [2]

static int [ ] = {10, 20, 30, 40, 50, 60, 70, 80};

i) What is the meaning of (x+2)?

ii) What is the value of *x?

iii) What is the value of (*x + 2)?

iv) What is the value of *(x + 2)?

b) Construct the truth table for the Boolean function f in x, y, z which takes the value 0 if and only if at least two of the variables take the value 1. Obtain DNF and CNF of f. [4]

c) Write a program in C to evaluate f(x) for x = 0, 2, 4, 6, ... 20 where

$f(x) = x^3 + 2\sin x$ if $x < 2$

$= 2x + 3$ if $x = 2$

$= e^x + \tan x$ if $x > 2$ [4]

11. a) Let $(B,+,\bullet,')$ be a Boolean algebra & $a,b,c \in B$. Prove that $a + b = a + c$ & $a \cdot b = a \cdot c$ implies $b = c$. [3]

b) Write a C program to determine the standard deviation of the following data 1·5, 3·8, 9·7, 4·18, 2·1. [3]

c) Write a C program that uses the algorithm of nested multiplication to find the value of the polynomial $2x^5 - 8x^4 + 3\cdot 5x^3 + x^2 - 20x + 9\cdot 8$ when x = 1·08. [4]

Group – C [Tensor Calculus]

(*Answer **any four** questions*) [4×5]

12. If B^{β}_{γ} be the components of a mixed tensor of rank 2 and if $A(\alpha,\beta,\eta)B^{\beta}_{\gamma} = C^{\alpha}_{\eta}$ then show that $A(\alpha,\beta,\eta)$ is a tensor. Determine its rank & type. [5]

13. a) If a tensor A_{ijk} is skew-symmetric in the first two indices from the left and symmetric in the second and third indices from the left, show that $A_{ijk} = 0$. [2]

b) Show that in the Riemannian space V_4 with line element

$(ds)^2 = -(dx^1)^2 - (dx^2)^2 - (dx^3)^2 + c^2(dx^4)^2$, the vector $\left(1,1,0,\frac{\sqrt{3}}{c}\right)$ is a unit vector. [2]

c) Prove that $[ij,k] + [kj,i] = \frac{\partial g_{ik}}{\partial x^j}$. [1]

14. Prove that $\left\{\begin{matrix} i \\ i\,j \end{matrix}\right\} = \frac{\partial}{\partial x^j}\log\sqrt{g}$ where $g = |g_{ij}| \neq 0$. [5]

15. If A_i is an arbitrary covariant vector & $C^{ij}A_iA_j$ is an invariant prove that $C^{ij} + C^{ji}$ is a contravariant tensor of rank 2. [5]

16. Prove that g_{ij} is a symmetric covariant tensor of rank 2. [5]

17. Prove that $\left(g_{ij}A^i\right)_{,k} = A_{j,k}$ where the symbols have their usual meaning. [5]

OR

Group – C [Differential Geometry]

(*Answer **any four** questions*) [4×5]

18. Compute the first and second fundamental form of the surface patch $\sigma(u,v) = (u\cos v, u\sin v, cv)$, 'c' being a constant, Hence, find the mean curvature of this surface. What geometrical conclusion can you make from the result? [1+3+1]

19. Let $\gamma(t) = (\cos^2 t, \sin^2 t)$ for $t \in \left(0,\frac{\pi}{2}\right)$ be a curve in $\mathbb{R}^2$.

a) Is it regular? [1]

b) Find the unit speed reparametrigation of γ. [2]

c) Find the curvature of γ at $t=\frac{\pi}{3}$. [2]

20. Let $\Phi:(a,b)\to\mathbb{R}$ be a smooth function where $a,b\in\mathbb{R}$, $a<b$. Prove that there exists a unique (upto a rigid motion) unit speed curve $\gamma:(a,b)\to\mathbb{R}^2$ whose signed curvature is Φ. [5]

21. Prove that for a space curve, $\frac{\partial\mu^i}{\partial s}=-R\lambda^i+\tau\gamma^i$ where $(\lambda^i,\mu^i,\gamma^i)$ are components of unit tagent, principal normal and binormal vectors and (R,τ) are curvature and torsion of the space curve at some point, 's' being the arc length parameter. [5]

22. Let S_1, S_2 be two regular smooth surfaces and $f:S_1\to S_2$ be a diffeomorphism. Prove that for each σ_2, an allowable surface patch on S_2, $f^{-1}o\,\sigma_2$ is an allowable surface patch on S_1. [5]

23. a) Find the normal curvature of the circle $\gamma(t)=(1,\cos t,\sin t)$ on the elliptic paraboloid $\sigma(u,v)=(u^2+v^2,u,v)$. [4]

    b) Define principal curvatures of a surface patch. [1]

OR

Group – C [Topology]

(*Answer **any four** questions*) [4×5]

24. a) Let τ be the discrete topology on a set X. Describe all bases of (X,τ). [3]

    b) Let τ be the discrete topology on $\mathbb{R}$. Find all nonempty subsets in $(\mathbb{R},\tau)$ which are dense in $\mathbb{R}$. [2]

25. a) Show that a countable space with cofinite topology is first countable. [2]

    b) Suppose X is a 2^{nd} countable space and A is an uncountable subset of X. Show that $A\cap A^d\neq\phi$. [3]

26. Give an example with justification of a topological space which is first countable, separable, Lindelöf but not second countable. [5]

27. a) Prove that in a topological space the union of any subfamily of a locally finite family of closed sets is closed.

    [A family of subsets of a topological space is called locally finite if each point of the space has an open neighbourhood intersecting only finitely many members of the family] [2]

    b) Give an example with justification of a bijective continuous map which is not a homeomorphism. [3]

28. Prove that a closed subspace of a Lindelöf space is Lindelöf. Give an example to show that the Lindelöf property is not hereditary. [3+2]

29. Let (X,τ) be a topological space, $A\subset X$ and $x_0\in X$. If there is a sequence $\{x_n\}_{n\in\mathbb{N}}$ in A converging to x_0, then prove that $x_0\in\bar{A}$. Does the converse of the above result always hold? Support your answer. [2+3]

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